

January 8, 1885.

THE TREASURER in the Chair.

The Presents received were laid on the table, and thanks ordered for them.

Dr. Morrison Watson was admitted into the Society.

In accordance with the announcement made from the Chair at the last Meeting, the question of Mr. James Bateman's re-admission was put to the vote. It was decided in the affirmative, whereupon the Treasurer declared that Mr. Bateman was re-admitted into the Society.

The following Papers were read :—

- I. "Experimental Researches in Magnetism." By Professor J. A. EWING, B.Sc., F.R.S.E., University College, Dundee. Communicated by Sir WILLIAM THOMSON, F.R.S. Received December 5, 1884.

(Abstract.)

The paper describes in detail experiments of which preliminary notices have already been published in the "Proceedings of the Royal Society," vol. 34, p. 39, and in the "Philosophical Magazine," November, 1883. The experiments relate to—

(1.) The magnetic susceptibility of iron and steel, the form of the magnetisation curve, and the changes of magnetism caused by cyclic changes of magnetising force.

(2.) The influence of vibration on magnetic susceptibility and retentiveness.

(3.) The influence of permanent strain on magnetic susceptibility and retentiveness.

Order," Arts. 53–58, it is shown that the general form of which the criticoid J_2 is a particular case, enables us to solve for all orders the problem of destroying the second and third terms, that is, it makes it depend on the solution of equations of the orders 1 and 2. For other applications of the theory, see Cockle "On Primary Forms," Arts. 6–14, "Philosophical Magazine" for February, 1875, vol. xlix, ser. 4, pp. 134–142. See also the "Mathematical Reprint from the Educational Times," vol. 34, pp. 109, 110; Question 6503.

(4.) The energy expended in producing cyclic changes of magnetisation.

(5.) The ratio of residual to total induced magnetism.

(6.) The changes of induced and residual magnetism caused by changes of stress.

(7.) The effects of constant stress on magnetic susceptibility and retentiveness.

(8.) The changes of magnetism caused by changes of temperature.

(9.) The effect of temperature on magnetic susceptibility.

The experiments were conducted on pieces of metal which gave as near an approach to the condition of uniform magnetisation as is practically attainable. In some cases rings were used in a manner similar to that employed by Stoletow and Rowland, except that in the present case the process of magnetisation was effected in a series of sudden steps, the total magnetisation being found by summing up the inductive effect of the steps. In most cases, however, the author used straight pieces of wire, whose length was 300 or 400 times their diameter. These permitted the use of a direct magnetometric method of observation, and consequently the magnetising force could be applied and removed continuously and slowly, instead of by sudden steps. Preliminary experiments had satisfied the author that wires of this length gave as near an approach to uniform magnetisation as could be practically obtained by using rings.

Curves are given which show the behaviour of iron and steel in various states of temper, when subjected to a first application of magnetising force, and also to subsequent cyclic changes of magnetising force, such as complete or partial removal and re-application, or reversal. The curves are drawn by plotting either $\mathfrak{J}$, the intensity of magnetisation, or $\mathfrak{B}$, the magnetic induction, in relation to $\mathfrak{H}$, the magnetising force: the characteristics of these curves and their relation to the physical state of the piece under examination are pointed out. Curves so drawn invariably exhibit the static lagging action to which the author (in a former paper) gave the name "hysteresis," any cyclic change of $\mathfrak{H}$ giving rise to a more or less nearly closed *loop* in the curve. Attention was previously drawn to these loops by Warburg, who also anticipated the author in pointing out their important physical meaning, namely, that the area of a loop, or $-\int \mathfrak{J} d\mathfrak{H}$, is the measure of the energy expended (per unit of volume) in performing the cycle of magnetisation which the loop describes. In the present paper numerous absolute measurements of this energy are given, especially of the energy which is thus dissipated in each reversal of the magnetism of a piece of iron or steel. These show that while the dissipation of energy by reversal of magnetism is very much smaller in soft iron than in hard iron or steel, even in the latter its amount is very trifling, so that the principal part of the heat which is produced in the cores of

electromagnets must be due chiefly to other causes than this static hysteresis, and is, in fact, due almost wholly to the induction of so-called Foucault currents in the cores. The relation of this hysteresis to Weber's theory of molecular magnets, as extended by Maxwell, is discussed, and the insufficiency of Maxwell's extension noticed.

By vibrating a piece of soft iron during the application and removal of magnetising force, the effects of hysteresis are almost entirely removed, and the iron is then found to possess almost no retentiveness. But when the application and removal of magnetising force are effected without mechanical disturbance, the retentiveness of soft iron is found to be even greater than that of steel. In some cases 93 per cent. of the whole induced magnetism of a piece of annealed iron was found to remain, on the complete removal of the magnetising force. It is pointed out that there is no discrepancy between this result and the well-known fact that a *short* iron core of an electromagnet retains almost no magnetism when the current in the magnet is interrupted. In that case the ends of the magnet itself, after the interruption of the current, exert a sufficient reversed magnetising force to destroy almost entirely the residual magnetism. But when tested under the conditions which give uniform magnetisation and avoid the demagnetising influence of the ends, soft annealed iron is more retentive than even the hardest steel.

Examples are given showing that the influence of permanent set in the curve of magnetisation is so marked as to give a criterion by which a strained piece may be readily distinguished from an annealed piece of metal, and that strain diminishes very greatly the magnetic retentiveness of iron.

Numerical values of the coefficients of permeability (μ) and of susceptibility (κ) are given for a number of samples of iron and steel, and the relation of these coefficients to $\mathfrak{B}$ and $\mathfrak{H}$ is exhibited graphically after the manner of Rowland. The greatest value of μ refers to soft annealed iron while under mechanical vibration, and is about 20,000.

The next part of the paper deals with the effects of stress (consisting of longitudinal pull) on the magnetic susceptibility and retentiveness of iron, and the subject is taken up in two different ways. In one the magnetic effects are observed when a load hanging from the wire under examination is gradually applied, removed, and otherwise varied, the wire being then either kept in a magnetising field of constant value, or freed from all magnetising force. In the other method the load is kept constant, and magnetising force is applied, removed, and otherwise varied. The two methods give results which would be identical were it not for (1) hysteresis in the relation of magnetism to load, in a constant field, and (2) hysteresis in the relation of magnetism to field, under constant load. The relation of mag-

netism to varying load in a constant field is examined at great length, both in soft annealed wire, and in wire hardened by stretching, and the effects, which are very complex, are exhibited graphically. They also afford an easy criterion by which a stretched piece can be distinguished from an annealed piece, and, both in stretched and annealed metal, they exhibit static hysteresis, which can be more or less completely wiped out by mechanical vibration. The connexion of the author's present work with the earlier results of Villari, and the extensive researches of Sir W. Thomson, is pointed out.

Numerous curves are given, showing the relation to $\frac{M}{H}$ to $\frac{M}{H}$ when the process of magnetisation is conducted while the piece under test is maintained in a constant state of stress. These show very clearly the position of what Thomson has called the "Villari critical point," or point at which the sign of the effect of stress on magnetism becomes reversed; and it is shown that this reversal is associated rather with a particular value of magnetisation than with a particular value of magnetising force, the same reversal being found to occur when the magnetism dealt with is wholly residual.

Instances are given of very curious molecular reminiscences of previous loads, which were found to exist even when the previous loading had been performed at a time when the metal was entirely destitute of magnetism, and although the loads whose effects were found to remain were much less than loads previously applied to the same wire, and were consequently unable to produce mechanical change of any ordinary kind. These residual effects of former loads became visible on the next magnetisation of the piece. It is shown that these effects of previous loads can be almost obliterated by subjecting the wire to vibration before beginning to magnetise it.

The ratio of residual to induced magnetism in soft and hardened iron and steel has been examined for various degrees of magnetisation, with the result of showing that the ratio passes a maximum when the magnetisation is moderately strong, and then falls off as the state of so-called saturation is reached. The influence of the presence of stress on this ratio and on residual magnetism generally has also been investigated.

The last part of the paper deals more briefly with the effect of temperature on magnetism, a subject already largely treated by G. Wiedemann and others. The present experiments were specially directed to discover whether there is hysteresis in the relation of magnetism to temperature when the temperature suffers cyclic changes. Careful experiments on both induced and residual magnetism have led the author to conclude that within the range of temperature dealt with by him, there is no perceptible hysteresis in the relation of magnetism to temperature. Curves are also given exhibiting the process of magnetising the same piece at different temperatures. These cross each

other in the same manner as the corresponding curves for different values of stress, a fact to be anticipated from the earlier discoveries of Baur.

The experiments, which have been of a very extended character, were made, during 1881-1883, in the Laboratory of the University of Tokio, Japan, with the help of Japanese students, Messrs. Fujisawa, Tanakadate, Tanaka, and Sakai, to whom the author is indebted for much valuable assistance. The results have been, almost without exception, reduced to absolute measure, and are for the most part presented graphically in curves which accompany the paper.

II. "On certain Definite Integrals. No. 12." By W. H. L. RUSSELL, F.R.S. Received December 8, 1884.

The following theorem must be implicitly known, although I have never seen it in print:—

$$\begin{aligned} \text{Let } \xi &= \lambda x + \mu y + \nu z, & \Delta x &= \Lambda \xi + M \eta + N \zeta. \\ \eta &= \lambda' x + \mu' y + \nu' z, & \Delta y &= \Lambda' \xi + M' \eta + N' \zeta. \\ \zeta &= \lambda'' x + \mu'' y + \nu'' z, & \Delta z &= \Lambda'' \xi + M'' \eta + N'' \zeta. \end{aligned}$$

$$\begin{aligned} \text{Then } \iiint dx dy dz \phi(\lambda x + \mu y + \nu z, \lambda' x + \mu' y + \nu' z, \lambda'' x + \mu'' y + \nu'' z) \\ = \iiint \Theta \phi(\xi, \eta, \zeta) d\xi d\eta d\zeta, \end{aligned}$$

where the limits of the first integral, and consequently those of the second, are given by an equation of limits, and Θ is a well-known constant. Now let

$$P = ax^3 + by^3 + cz^3 + a_1x^2y + a'_1x^2z + b_1y^2x + b'_1y^2z + c_1z^2x + c'_1z^2y + mxyz,$$

and let the expression break up into three linear factors, so that

$$P = (\lambda x + \mu y + \nu z)(\lambda' x + \mu' y + \nu' z)(\lambda'' x + \mu'' y + \nu'' z),$$

which will subject the constants $a, b, a_1, \&c.$, to three conditions.

$$\begin{aligned} \text{Then we have } \iiint \frac{dx dy dz x^{l-1} y^{m-1} z^{n-1}}{\sqrt{P}} = \\ \Sigma \frac{\rho \Delta^3}{(\Lambda + \Lambda' + \Lambda'')(M + M' + M'')(N + N' + N'')} \frac{\Gamma\left(\alpha - \frac{1}{r}\right) \Gamma\left(\beta - \frac{1}{r}\right) \Gamma\left(\gamma - \frac{1}{r}\right)}{\Gamma\left(1 + \alpha + \beta + \gamma - \frac{3}{r}\right)}, \end{aligned}$$

where l, m, n are positive integers, ρ a function of $\lambda, \mu, \nu, \lambda', \&c.$, which is different for each term, and $\alpha + \beta + \gamma = l + m + n$. The limits of the integral are given by the equation $x + y + z = 1$.